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Generalized Renormalization Group Equation for Systems of Arbitrary Symmetry

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An exact renormalization group equation for the Ginzburg–Landau–Wilson functional of an arbitrary symmetry is obtained. The equation derived does not contain redundant operators which must be transformed away.

Renormalization group (RG) theory has made tremendous progress in studying the singular behavior near the critical point of systems which undergo a continuous phase transition (for review see, e.g., Ma, 1976; Amit, 1984; Baker, 1990). There are a number of quite different approaches in the theory. One of the most fundamental is the approach based on the exact RG equation (Wilson and Kogut, 1974), which gives general insight into the structure of the theory. Although usually difficult to work with, this approach provides the basis for the theory of critical phenomena and may generate new approximation schemes (Golner and Reidel, 1976; Reidel *et al.*, 1985; Golner, 1986; Ivanchenko *et al.*, 1990; Lisyansky *et al.*, 1992). The substantial drawback of this approach has been that an exact RG equation contains an infinite number of redundant operators (Wegner and Hougton, 1973; Bell and Wilson, 1974; Wegner, 1976) which carry no physical meaning and should therefore be eliminated. For isotropic systems this problem was solved in Ivanchenko and Lisyansky (1992) and Ivanchenko *et al.* (1992), where the exact RG equation that does not contain redundant operators was obtained. This equation made it possible to develop a new perturbation theory using the numerically small critical exponent η as an expansion parameter (Ivanchenko *et al.*, 1992). Ivanchenko and Lisyansky (1992) and Ivanchenko *et al.* (1992) considered isotropic systems only. In the present paper, we substantially generalize the scheme developed there and obtain

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the exact renormalization group equation free of redundant operators for a system of an arbitrary symmetry.

We consider a system described by a Ginzburg–Landau–Wilson functional of a very general type,

$$H_I[\phi(\mathbf{q})] = \sum_{k=0}^{\infty} 2^{1-2k} \int_{q_1 \cdots q_{2k}} \sum_{\alpha_1, \ldots, \alpha_{2k}=1}^{n} \times \left[g_k^{\alpha_1, \ldots, \alpha_{2k}}(\mathbf{q}_1, \ldots, \mathbf{q}_{2k})(2\pi)^d \delta\left(\sum_{i=1}^{2k} \mathbf{q}_i\right) \prod_{i=1}^{2k} \phi^{\alpha_i}(\mathbf{q}_i)\right] \quad (1)$$

where ϕ is an n-component vector, $\int_q \equiv \int d^d q/(2\pi)^d$. The vertices $\hat{g}_k$ have an arbitrary tensorial structure with respect to indices α_i but possess an obvious symmetry

$$g^{\ldots\alpha_i\ldots\alpha_j\ldots}(\ldots, \mathbf{q}_i, \ldots, \mathbf{q}_j, \ldots) = g^{\ldots\alpha_j\ldots\alpha_i\ldots}(\ldots, \mathbf{q}_j, \ldots, \mathbf{q}_i, \ldots) \quad (2)$$

All momentum integrals in equation (1) must be cut off on some momentum scale Λ. It is much more convenient to perform all momentum integrations up to infinity, but instead we add to the functional (1) a term H_0 that provides a momentum cutoff for the theory,

$$H_0[\phi] = \frac{1}{2} \int_q G_0^{-1}(q, \Lambda) |\phi(\mathbf{q})|^2 \quad (3)$$

where the propagator G_0 is defined by

$$G_0(q, \Lambda) = q^{-2} S(q^2/\Lambda^2) \quad (4)$$

Here $S(x)$ is a monotonic function with the properties $S(x=0)=1$ and $\lim_{x\to\infty} S(x)x^m = 0$ for any m. Assuming that the vertices $\hat{g}_k(\mathbf{q}_1, \ldots, \mathbf{q}_{2k})$ do not diverge with increasing $\mathbf{q}_i$, then the term H_0 provides either a smooth cutoff when $S(x)$ is a smooth function or a sharp cutoff when $S(x)$ is a step function.

We now perform the following two steps, which are standard for the RG theory. First, we decrease the number of degrees of freedom of the sytem described by the functional $H[\phi] = H_0[\phi] + H_I[\phi]$ by integrating the partition function with respect to Fourier components $\phi(\mathbf{q})$ with momenta within a spherical shell $\Lambda(1-\xi) < q < \Lambda$ in momentum space with $\xi \ll 1$. Second, we rescale all momenta in order to restore the original momentum cutoff Λ. To do this, we represent the partition function in the form

$$Z = \int D\phi \exp(-H[\phi]) = Z_0 \langle \exp(-H_I[\phi]) \rangle_{0,\Lambda} \equiv Z_0 \langle w[\phi] \rangle_{0,\Lambda} \quad (5)$$

where

$$Z_0 = \int D\phi \exp(-H_0[\phi]) \quad (6)$$

and the averaging $\langle \cdots \rangle_{0,\Lambda}$ is performed with respect to the Gaussian functional $H_0[\phi]$ at a given value of Λ.

Before performing RG transformations, we write down the following identity:

$$\langle w[\phi] \rangle_{0,\Lambda} \equiv Z_0^{-1} \int D\phi \, w[\phi] \exp(-H_0[\phi])$$
$$= Z_{01}^{-1} Z_{02}^{-1} \int D\phi_1 D\phi_2 w[\phi_1, \phi_2] \exp(-H_0[\phi_1, \phi_2]) \quad (7)$$

where

$$Z_{0i} = \int D\phi_i \exp\left[-\frac{1}{2} \int_q G_{0i}^{-1}(q, \Lambda_i) |\phi_i(\mathbf{q})|^2 \right]$$
$$H_0[\phi_1, \phi_2] = \frac{1}{2} \int_q G_{01}^{-1}(q, \Lambda_1) |\phi_1(\mathbf{q})|^2 + \frac{1}{2} \int_q G_{02}^{-1}(q, \Lambda_2) |\phi_2(\mathbf{q})|^2 \quad (8)$$

with $\phi(\mathbf{q}) = \phi_1(\mathbf{q}) + \phi_2(\mathbf{q})$ and $G_0(q, \Lambda) = G_{01}(q, \Lambda_1) + G_{02}(q, \Lambda_2)$. If we now assume that $G_{01}(q, \Lambda_1) = G_0(q, \Lambda(1 - \xi))$ with $\xi \ll 1$, then $\phi_2(\mathbf{q})$ are the modes with momenta within a shell $\Lambda(1 - \xi) < q < \Lambda$ which should be integrated out. In this case G_{02} is given by

$$G_{02}(q, \Lambda_2) = G_0(q, \Lambda) - G_{01}(q, \Lambda_1) \simeq \xi \Lambda \frac{\partial G_0(q, \Lambda)}{\partial \Lambda} \equiv 2\xi h(q)$$
$$h(q) = q^{-2} \Lambda^2 \frac{dS(q^2/\Lambda)}{d\Lambda^2} \quad (9)$$

In order to perform the integration with respect to short-wave modes, we expand $\langle w[\phi_1, \phi_2] \rangle$ with respect to $\phi_2(\mathbf{q})$ in equation (7) so that

$$\langle w[\phi] \rangle_{0,\Lambda} = Z_{01}^{-1} Z_{02}^{-1} \int D\phi_1 \, D\phi_2 \left[w[\phi_1] + \int_q \sum_\alpha \frac{\delta w[\phi_1]}{\delta \phi_1^\alpha(\mathbf{q})} \phi_2^\alpha(\mathbf{q}) \right.$$
$$\left. + \frac{1}{2} \int_{qq'} \sum_{\alpha,\beta} \frac{\delta^2 w[\phi_1]}{\delta \phi_1^\alpha(\mathbf{q}) \delta \phi_1^\beta(\mathbf{q}')} \phi_2^\alpha(\mathbf{q}) \phi_2^\beta(\mathbf{q}') + \cdots \right]$$
$$\times \exp(-H_0[\phi_1, \phi_2]) \quad (10)$$

Since $H_0[\phi_1, \phi_2]$ is a quadratic form with respect to ϕ_2 only, even terms of the expansion (10) survive. Keeping terms of lowest order with respect to ξ only, we find

$$\langle w[\phi(\mathbf{q})] \rangle_{0,\Lambda} \simeq \langle (1 + \xi \hat{L}_A) w[\phi(\mathbf{q})] \rangle_{0,\Lambda(1-\xi)} \quad (11)$$

where the operator $\hat{L}_A$ is defined as

$$\hat{L}_A = V \int_q h(q) \sum_\alpha \frac{\delta^2}{\delta\phi^\alpha(\mathbf{q})\delta\phi^\alpha(-\mathbf{q})} \tag{12}$$

and the averaging $\langle\cdots\rangle_{0,\Lambda(1-\xi)}$ is performed with the functional

$$H_0[\phi, \Lambda(1-\xi)] = \frac{1}{2}\int_q G_0^{-1}\langle q, \Lambda(1-\xi)\rangle|\phi(\mathbf{q})|^2 \tag{13}$$

Therefore, the right-hand side of equation (11) contains effectively only modes with $q < \Lambda(1-\xi)$. This completes the first step of the RG transformation.

For the second step we restore the momentum cutoff Λ through the transformation $\mathbf{q} = \mathbf{q}'(1-\xi)$. This transformation, however, does not restore the original functional H_0,

$$\begin{aligned} H_0[\phi, \Lambda(1-\xi)] &= \frac{1}{2}\int_q G_0^{-1}(q, \Lambda(1-\xi))|\phi(\mathbf{q})|^2 \\ \to H_0'[\phi, \Lambda] &= \frac{(1-\xi)^{d+2}}{2}\int_q G_0^{-1}(q, \Lambda)|\phi(\mathbf{q}(1-\xi))|^2 \end{aligned} \tag{14}$$

To transform $H_0'(\Lambda)$ into $H_0(\Lambda)$, we make the substitution

$$\begin{aligned} \phi^\alpha(\mathbf{q}) &= \sum_\beta [\delta^{\alpha\beta} + \xi\varepsilon^{\alpha\beta}(\mathbf{q})]\phi'^\beta(\mathbf{q}(1+\xi)) \\ &= \sum_\beta \left[\delta^{\alpha\beta} + \xi\left(\varepsilon^{\alpha\beta}(\mathbf{q}) + \delta^{\alpha\beta}\,\mathbf{q}\cdot\frac{\partial}{\partial\mathbf{q}}\right)\right]\phi'^\beta(\mathbf{q}) \end{aligned} \tag{15}$$

where at present $\varepsilon^{\alpha\beta}(\mathbf{q})$ is an arbitrary function with the only condition $\varepsilon^{\alpha\beta}(\mathbf{q}) = \varepsilon^{\beta\alpha}(-\mathbf{q})$, which preserves the symmetry (2). Applying the transformation (15) to $\langle w[\phi]\rangle_{0,\Lambda(1-\xi)}$ in equation (11) and keeping terms of lowest order in ξ, we obtain

$$\langle w'[\phi]\rangle_{0,\Lambda} = \langle[1 + \xi(\hat{L}_A + \hat{L}_B + \hat{L}_C + \hat{L}_V)]w[\phi]\rangle_{0,\Lambda} \tag{16}$$

where the operator $\hat{L}_A$ is defined by (12), $\hat{L}_B$ is a result of the expansion (15),

$$\hat{L}_B \equiv \int_q \sum_{\alpha,\beta}\left[\varepsilon^{\alpha\beta}(-\mathbf{q}) + \delta^{\alpha\beta}\,\mathbf{q}\cdot\frac{\partial}{\partial\mathbf{q}}\right]\phi^\alpha(\mathbf{q})\frac{\delta}{\delta\phi^\beta(\mathbf{q})} \tag{17}$$

$\hat{L}_C$ arises from the transformation of $H_0(\Lambda(1-\xi))$ into $H_0(\Lambda)$,

$$\begin{aligned} \hat{L}_C = &\int_q G_0^{-1}(q, \Lambda)\sum_{\alpha,\beta}\left[\delta^{\alpha\beta}\frac{d+2}{2} - \varepsilon^{\alpha\beta}(-\mathbf{q})\right]\phi^\alpha(\mathbf{q})\phi^\beta(-\mathbf{q}) \\ &- V\int_q \sum_\alpha\left[\frac{d+2}{2} - \varepsilon^{\alpha\alpha}(\mathbf{q})\right] \end{aligned} \tag{18}$$

and $\hat{L}_V$ comes about when the shrinking of volume $V = V'(1+\xi d)$ is considered,

$$\hat{L}_V = Vd\frac{\partial}{\partial V} \tag{19}$$

At this point one can derive the final RG equation for the functional H_I by using the relationship $w[\phi] = \exp(-H_I[\phi])$,

$$\begin{aligned}\dot{H}_I[\phi] &= Vd\frac{\partial H_I[\phi]}{\partial V} + \frac{V}{2}\int_q \sum_\alpha \eta^{\alpha\alpha}(\mathbf{q}) - \frac{1}{2}\int_q \sum_{\alpha,\beta} \eta^{\alpha\beta}(\mathbf{q}) G_0^{-1}(q,\Lambda)\phi^\alpha(\mathbf{q})\phi^\beta(-\mathbf{q}) \\ &+ \int_q \sum_{\alpha,\beta}\left[\left(\frac{d+2}{2}\delta^{\alpha\beta} - \frac{\eta^{\alpha\beta}(\mathbf{q})}{2}\right)\phi^\alpha(\mathbf{q}) + \delta^{\alpha\beta}\mathbf{q}\cdot\frac{\partial\phi^\alpha(\mathbf{q})}{\partial\mathbf{q}}\right]\frac{\delta H_I[\phi]}{\delta\phi^\beta(q)} \\ &+ \int_q h(q)\sum_\alpha\left[\frac{\delta^2 H_I[\phi]}{\delta\phi^\alpha(\mathbf{q})\delta\phi^\alpha(-\mathbf{q})} - \frac{\delta H_I[\phi]}{\delta\phi^\alpha(\mathbf{q})}\frac{\delta H_I[\phi]}{\delta\phi^\alpha(-\mathbf{q})}\right]\end{aligned} \tag{20}$$

Here we defined $\eta^{\alpha\beta}(\mathbf{q})$ as

$$\eta^{\alpha\beta}(\mathbf{q}) = \delta^{\alpha\beta}(d+2) - 2\varepsilon^{\alpha\beta}(-\mathbf{q}) \tag{21}$$

Equation (20) is an exact RG equation for the anisotropic functional (1). This equation contains an arbitrary function $\eta^{\alpha\beta}(\mathbf{q})$, which has to be defined. The most direct choice for $\eta^{\alpha\beta}(\mathbf{q})$ seems to be zero. This choice would simplify the equation and make it similar to the traditional ones. However, such an equation contains redundant operators, which would have to be eliminated by developing a proper procedure. Our goal is to obtain an equation free of redundant operators. This can be achieved by a special choice of the function $\eta^{\alpha\beta}(\mathbf{q})$. First of all, let us note that within the renormalization procedure the vertex $\hat{g}_1(\mathbf{q})$ is renormalized. The whole or a portion of the q-dependent part of this renormalization can be incorporated into the function G_0^{-1} of the functional H_0. This would change the cutoff procedure, which should, however, remain the same within the renormalization process. To avoid this, we define the function $\eta^{\alpha\beta}(\mathbf{q})$ such that it cancels the q-dependent renormalization of the vertex $\hat{g}_1(\mathbf{q})$. In order to do this, we extract an explicit equation for this vertex from equation (20).

$$\begin{aligned}\dot{g}_1^{\alpha\beta}(\mathbf{q}) &= -\eta^{\alpha\beta}(\mathbf{q})G_0^{-1}(q,\Lambda) + \sum_\gamma\left[2\delta^{\alpha\gamma} - \eta^{\alpha\gamma}(\mathbf{q}) - \delta^{\alpha\gamma}\mathbf{q}\cdot\frac{\partial}{\partial\mathbf{q}}\right]g_1^{\gamma\beta}(\mathbf{q}) \\ &+ \sum_\gamma Q^{\alpha\beta\gamma\gamma}(\mathbf{q}) - 2\sum_\gamma g_1^{\alpha\gamma}(\mathbf{q})g_1^{\gamma\beta}(\mathbf{q})h(q)\end{aligned} \tag{22}$$

where

$$Q^{\alpha\beta\gamma\delta}(\mathbf{q}) = 3\int_p h(p)g_2^{\alpha\beta\gamma\delta}(\mathbf{q},-\mathbf{q},\mathbf{p},-\mathbf{p}) \tag{23}$$

We can now split equation (22) into two equations: one for $\hat{g}_{10}$, which is the momentum-independent part of $\hat{g}_1(\mathbf{q})$, and another for $\hat{g}'_1(\mathbf{q}) = \hat{g}_1(\mathbf{q}) - \hat{g}_{10}$,

$$\dot{g}_{10}^{\alpha\beta} = \sum_\gamma [2\delta^{\alpha\gamma} - \eta^{\alpha\gamma}(0)]g_{10}^{\gamma\beta} + \sum_\gamma Q^{\alpha\beta\gamma\gamma}(0) - 2\sum_\gamma g_{10}^{\alpha\gamma}g_{10}^{\gamma\beta}h(0) \tag{24}$$

$$\begin{aligned}\dot{g}_1'^{\alpha\beta}(\mathbf{q}) = -\eta^{\alpha\beta}(\mathbf{q})G_0^{-1}(q,\Lambda) + \sum_\gamma \Bigg\{ &\left[2\delta^{\alpha\gamma} - \eta^{\alpha\gamma}(\mathbf{q}) - \delta^{\alpha\gamma}\mathbf{q}\cdot\frac{\partial}{\partial\mathbf{q}}\right]g_1'^{\gamma\beta}(\mathbf{q}) \\ &- [\eta^{\alpha\gamma}(\mathbf{q}) - \eta^{\alpha\gamma}(0)]g_{10}^{\gamma\beta} + Q^{\alpha\beta\gamma\gamma}(\mathbf{q}) - Q^{\alpha\beta\gamma\gamma}(0) - 2g_{10}^{\alpha\beta}g_{10}^{\gamma\beta}[h(q) - h(0)] \\ &- 2[g_{10}^{\alpha\gamma}g_1'^{\gamma\beta}(\mathbf{q}) + g_1'^{\alpha\gamma}(\mathbf{q})g_{10}^{\gamma\beta} + g_1'^{\alpha\gamma}(\mathbf{q})g_1'^{\gamma\beta}(\mathbf{q})]h(q)\Bigg\}\end{aligned} \tag{25}$$

Using equation (25), we can define the function $\eta^{\alpha\beta}(\mathbf{q})$ such that the derivative of $\hat{g}'_1(\mathbf{q})$ is equal to zero. This means that if the vertex $\hat{g}_1(\mathbf{q})$ of the initial functional H_I is constant, then a q-dependent part of this vertex will not be generated and the functional H_0 will be intact within the renormalization procedure. The requirement $\hat{g}'_1(\mathbf{q}) = 0$ implies that

$$\begin{aligned}\eta^{\alpha\beta}(\mathbf{q}) = \eta^{\alpha\beta}(0) - \sum_\nu \Bigg(&\eta^{\alpha\nu}(0)G_0^{-1}(g,\Lambda) + \sum_\gamma \{2[h(q) - h(0)]g_{10}^{\alpha\gamma}g_{10}^{\gamma\nu} \\ &- Q^{\alpha\nu\gamma\gamma}(\mathbf{q}) + Q^{\alpha\nu\gamma\gamma}(0)\}\Bigg)[g_{10}^{\nu\beta} + G_0^{-1}(q,\Lambda)\delta^{\nu\beta}]^{-1}\end{aligned} \tag{26}$$

Equation (26) defines the momentum-dependent part of the function $\eta^{\alpha\beta}(\mathbf{q})$. We still have to define n^2 components of the tensor $\eta^{\alpha\beta}(0)$. We can use these values to simplify the RG equations and clarify the physical meaning of $\eta^{\alpha\beta}$. To do this, let us diagonalize the vertex $\hat{g}_{10}$ in the initial functional H_I. This can always be done without loss of generality. The diagonal components of this tensor are trial critical temperatures for the corresponding components of the order parameter $\phi(\mathbf{q})$. However, as one can see from equation (24), even if the nondiagonal part of the tensor $g_{10}{}^{\alpha\beta}$ does not exist in the initial functional, it will be generated within the renormalization process. We can use the arbitrariness of tensor $\eta^{\alpha\beta}$ to keep the tensor $g_{10}{}^{\alpha\beta}$ diagonal. In order to do this, let us split equation (24) into two separate equations for diagonal and nondiagonal parts of the vertex $g_{10}{}^{\alpha\beta}$. Defining $g_{10}{}^{\alpha\beta} = \delta^{\alpha\beta}r^\alpha + (1 - \delta^{\alpha\beta})r^{\alpha\beta}$, we have

$$\dot{r}^\alpha = (2 - \eta^\alpha)r^\alpha + Q^\alpha(0) + 2(r^\alpha)^2h(0) \tag{27}$$

$$\begin{aligned}\dot{r}^{\alpha\beta} = (2 - \eta^\alpha)r^{\alpha\beta} - \sum_\gamma \tilde{\eta}^{\alpha\gamma}r^{\gamma\beta} + \tilde{Q}^{\alpha\beta}(0) \\ - \tilde{\eta}^{\alpha\beta}r^\beta - 2\left[\left(r^\alpha r^{\alpha\beta} + r^{\alpha\beta}r^\beta + \sum_\gamma r^{\alpha\gamma}r^{\gamma\beta}\right)\right]h(0)\end{aligned} \tag{28}$$

where η^α, Q^α and $\tilde{\eta}^{\alpha\beta}$, $\tilde{Q}^{\alpha\beta}$ are diagonal and nondiagonal elements of tensors $\eta^{\alpha\beta}$ and $\sum_\gamma Q^{\alpha\beta\gamma\gamma}$, respectively,

$$\begin{aligned}\eta^{\alpha\beta}(0) &= \delta^{\alpha\beta}\eta^\alpha + (1-\delta^{\alpha\beta})\tilde{\eta}^{\alpha\beta}\\ \sum_\gamma Q^{\alpha\beta\gamma\gamma}(\mathbf{q}) &= \delta^{\alpha\beta}Q^\alpha(\mathbf{q}) + (1-\delta^{\alpha\beta})\tilde{Q}^{\alpha\beta}(\mathbf{q})\end{aligned} \tag{29}$$

Now, by choosing

$$\tilde{\eta}^{\alpha\beta} = \tilde{Q}^{\alpha\beta}/r^\beta \tag{30}$$

we provide that if the initial functional does not contain nondiagonal parts of the vertex $\hat{g}_1$, then this vertex remains diagonal after the renormalization. If at last we require that the expansion of $\eta^{\alpha\alpha}(\mathbf{q})$ does not contain q^2 terms, then the following equation defines the diagonal part of the tensor $\eta^{\alpha\beta}(0)$:

$$\eta^\alpha = \frac{d}{dq^2}[Q^\alpha(\mathbf{q}) - 2h(q)(r^\alpha)^2]_{q=0} \tag{31}$$

The function $\eta^{\alpha\beta}(\mathbf{q})$ is now completely defined and there is no more freedom in the exact RG equation (20); therefore, it must contain no redundant operators. The physical meaning of the function $\eta^{\alpha\beta}$ is suggested by the equation (27): at the stable fixed point of the functional (1), η^α is equal to the critical exponent η of the corresponding critical mode ϕ^α.

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